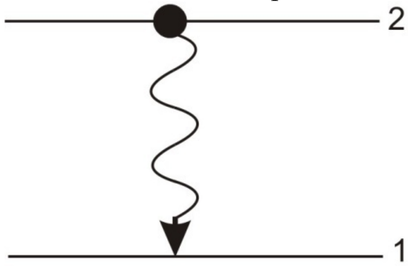
4061- Lecture Three

Two Level Atoms in Thermal Equilibrium

- Populations of levels remain constant
- A balance rate of upwards and downward transitions

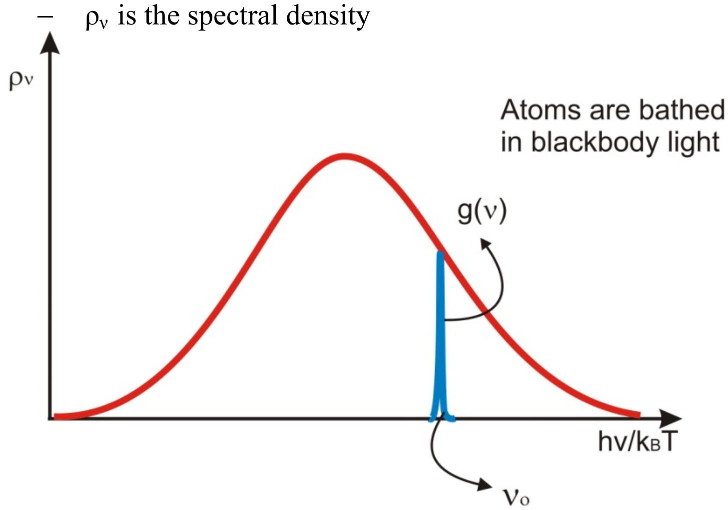


$n_2 = n_1 e^{-E_{21}/k_B T} = n_1 e^{-h\nu_o/k_B T}$ (1)

Blackbody or Plank Spectrum

$\rho_\nu = (8\pi h\nu^3/c^3)\left(\frac{1}{e^{\frac{h\nu_o}{k_B T}} - 1}\right)$ (2)

- ρ_ν is the spectral density



  - constant over atomic Lineshape g(ν)

$\frac{dn_2}{dt} = n_1 B_{12}\, \rho_\nu - n_2 B_{21}\, \rho_\nu - n_2 A_{12}$ (3)

- terms $B_{12}\, \rho_\nu$, $B_{21}\, \rho_\nu$, A_{21} have dimensions of Rate [s^{-1}]
- For a steady state/equilibrium $dn_2/dt = 0$ (4)
- Using (4), substitute (1) into (3), and solve for n_2/n_1

$e^{-\frac{h\nu_o}{k_B T}} = \frac{B_{12}\rho_\nu}{A_{21}} + B_{21}\, \rho_\nu$ (5)

- This is true for all temperatures
- A and B coefficients are independent of temperature as they are properties of the atom

For High Temperature

- Rayleigh Jean's Law → $\rho_\nu \sim k_B T$
- For High temperature $k_B T >> h\nu$ which implies ρ_ν = large
- In equation 5, this implies that $B_{21}\,\rho_\nu >> A_{21}$
- Therefore (5) becomes:

$$e^{-0} = 1 = B_{12}\,\rho_\nu / B_{21}\,\rho_\nu$$
$$\Rightarrow \quad B_{12} = B_{21}$$

- $B_{21} = B_{12}$, solve for ρ_ν in equation 5

$$\rho_\nu = \frac{A_{21}}{B_{21}}\left(\frac{1}{e^{\frac{h\nu_o}{k_B T}}-1}\right) \quad (6)$$

- Comparing this with Planks distribution (equation 2) a relationship between the spontaneous emission rate and the stimulated emission rate can be found

$$A_{21}/B_{21} = 8\pi h\nu_o^3 / c^3$$

- Higher A coefficient implies a higher B coefficient. So a transition with a strong decay rate can be a good laser transition

Interpretation

ρ_ν = Energy/(Vol)($\Delta\nu$) = [Modes/(Vol)($\Delta\nu$)] [Energy/Photon] [Photons/Mode]

$$\rho_\nu = [\beta_\nu]\,[h\nu_o]\,[\,\bar{N}\,]$$
$$\rho_\nu = (8\pi\nu_o^2/c^3)(h\nu_o)\left[\frac{1}{e^{\frac{h\nu_o}{k_B T}}} - 1\right]$$

- $1/(e^{h\nu_o/k_B T}-1)$ is the occupation number for bosons (photons)
- For fermions it is +1 in the denominator rather than -1

Stimulated Emission Rate

$$R_{21} = \rho_\nu\, B_{21} \quad (7)$$
$$\beta_\nu h\nu = 8\pi h\nu^3/c^3 = A_{21} / B_{21}$$

Solving for B_{21} and plugging into (7), it can be shown that

$$R_{21} = \bar{N} A_{21}$$
$$R_{21(\text{stim emission rate})} = \bar{N}\, R_{21(\text{spont emission rate})} \quad (8)$$