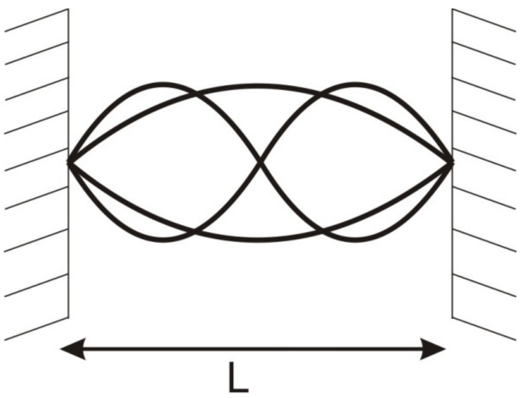
4061- Lecture Four

Electromagnetic Modes in 1D Cavity

ID Cavity assume Paraxial Fields

Mode Representation



Travelling Wave $E(x,t) = E_o \cos(kx - wt)$

- For constructive interference, $E(x + 2L, t) = E(x,t)$
- Standing wave in cavity = EM mode

Using boundary condition: $kx = m\pi$

$$kL = m\pi$$

Where m is an integer (mode index)

Since $k = 2\pi/\lambda$

$$L = m\lambda / 2$$

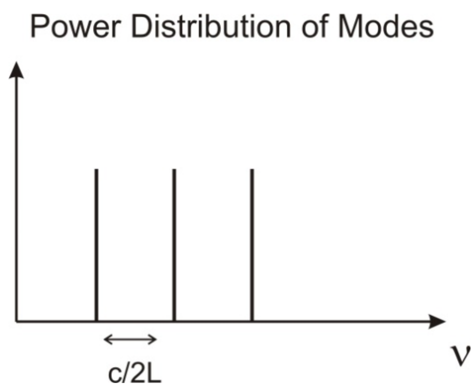
The condition $L = m\lambda/2$ means that an integer number of half wavelengths fit within the cavity

Mode Frequencies or Resonant Frequencies of the cavity are:

$$\nu_m = \frac{\frac{c}{n}}{\frac{2L}{m}} = m\frac{c}{2nL}$$

Here n is the refractive index of medium inside cavity.

For n=1, mode spacing is c/2L



Cavity Resonance Corresponds to Standing Wave

$$E_+(x,t) = E_o \cos(kx - wt + \phi)$$

$$E_-(x,t) = E_o \cos(kx + wt + \phi)$$

$$E = E_+ + E_- = 2E_o \cos(kx + \phi)\cos(wt)$$

Counting Modes in 3D Cavity Consisting of Cube of Side L

$$k_x = m_x\pi/L \qquad k_y = m_y\pi/L \qquad k_z = m_z\pi/L$$

Values represent spacing between discrete modes [grid]

Assume m_x, m_y, m_z, are positive since negative integers represent same modes as positive integers. This is because a given mode is associated with a combination of + and – waves.

- Number of modes up to frequency ν = number of modes with up to $k=2\pi\nu/c$
- Number of modes inside sphere of radius k :

$$N = \frac{4}{3}\pi k^3 \text{ x } \frac{1}{8} \text{ x } 2 \text{ x } \left[\frac{L}{\pi}\right]^3 = \frac{8\pi\nu^3 L^3}{3c^3}$$

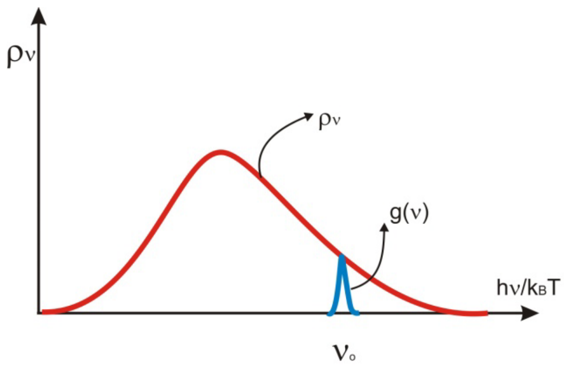
- 1/8 is the positive octant, 2 is the number of polarizations per mode, $\left[\frac{L}{\pi}\right]^3$ is the density of modes (number per unit volume)

Spectral Mode Density

$$\beta_\nu = \frac{\text{Modes}}{(\text{Vol})(\Delta\nu)} = 8\pi\nu^2/c^3$$

Comments about Stimulated Emission Rate

1. General Case



Spectral density uniform over atomic Lineshape

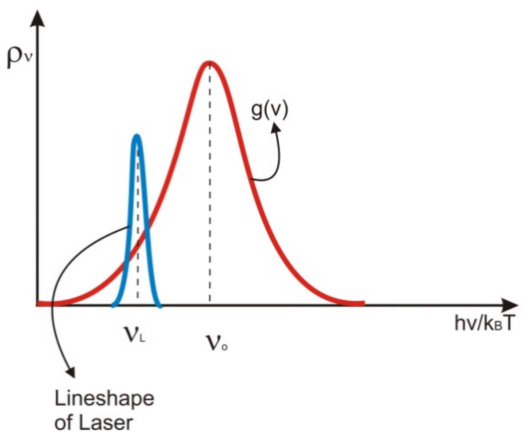
$R_{21} = \int \rho_\upsilon \, B_{21} g(\upsilon) d\upsilon = \rho_\upsilon(\nu = 0) B_{21} \int g(\upsilon) \, d\upsilon$

– $[\int g(\upsilon) \, d\upsilon] \rightarrow 1$ due to normalization

$$\mathbf{R_{21} = B_{21} P_{\nu o}}$$

– this is the implicit result for a black body

2. Precision Laser Spectroscopy



$R_{21} = \int \rho_\upsilon \, B_{21} g(\upsilon) d\upsilon = g(\upsilon_L) B_{21} \int \rho_\upsilon \, d\upsilon$

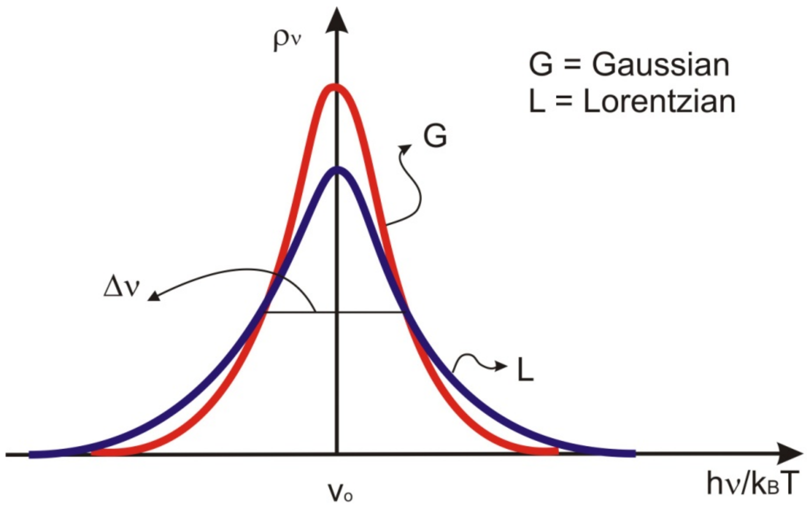
$R_{21} = g(\upsilon_L) B_{21} \rho$

– ρ = energy density = I/c associated with laser

$$\mathbf{R_{21} = g(\upsilon_L) B_{21} \left(\frac{I}{c}\right)}$$

– This is the result for the monochromatic case

Comments about Atomic Lineshape



Δν = full width half maximum

Gaussian Function:

$$g(\upsilon) = Ae^{-D(\upsilon-\upsilon_o)^2}$$

Example → Doppler Broadened Spectral line

$$A = \frac{1}{\Delta\upsilon}\sqrt{\frac{4\ln 2}{\pi}}$$

$$D = \frac{4\ln 2}{(\Delta\upsilon)^2}$$

$$g(\nu_o) = g_{max} = \frac{2}{\pi\Delta\nu}$$

Lorentzian Function:

$$L(\upsilon) = \frac{1}{\pi}\frac{\frac{\Delta\nu}{2}}{(\nu-\nu_o)^2 + (\frac{\Delta\nu}{2})^2}$$

Example → Natural Linewidth, Doppler Free Transition Lineshape, Power Broadened Lineshape

$$L(\nu_o) = L_{max} = \frac{2}{\pi\Delta\nu}$$