Physics 4062/5062 – Lecture Eleven –Optical Dipole Force – Classical Model

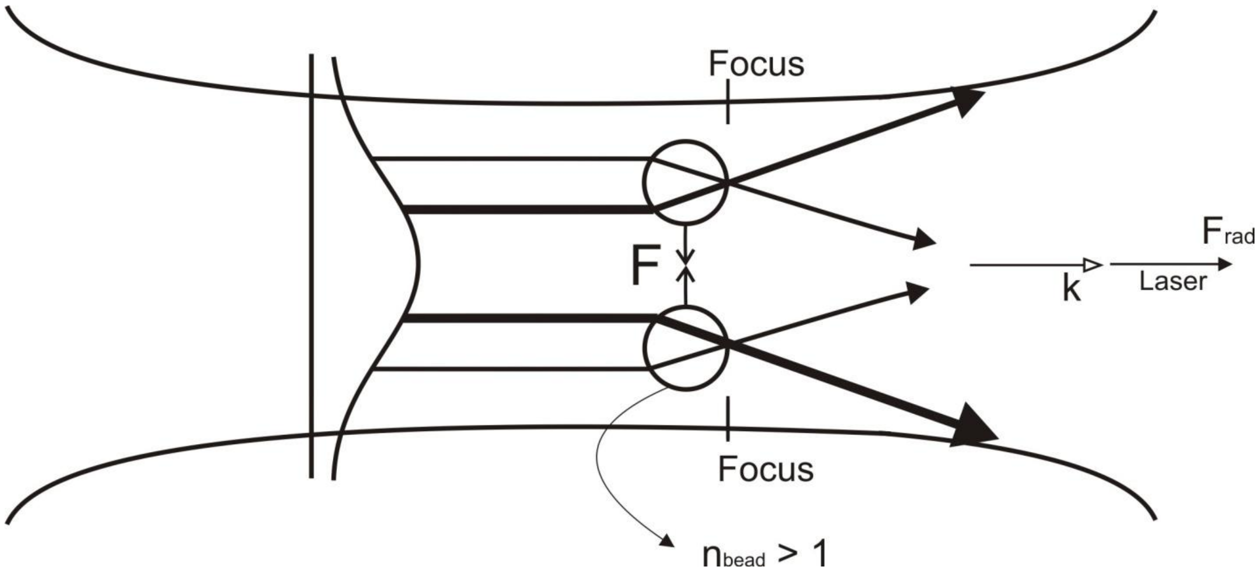
Outline

Describe Physical Origin of Optical Dipole Force

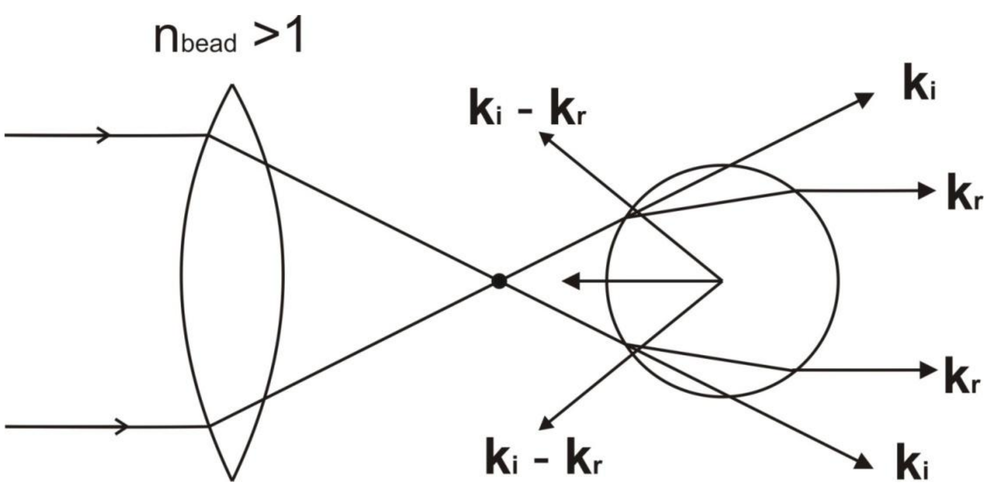
Using the classical model derive contributions to force arising from in phase and quadrature components of dipole moment.

The in phase component is associated with the optical dipole force and the index of refraction. The quadrature component is associated with the radiation pressure force and absorption.

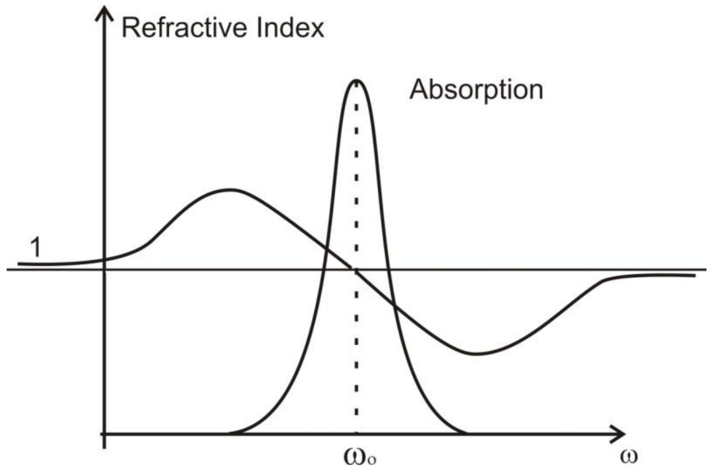
Optical Dipole Force (ODF) → used in optical tweezers- extensive applications in biophysics



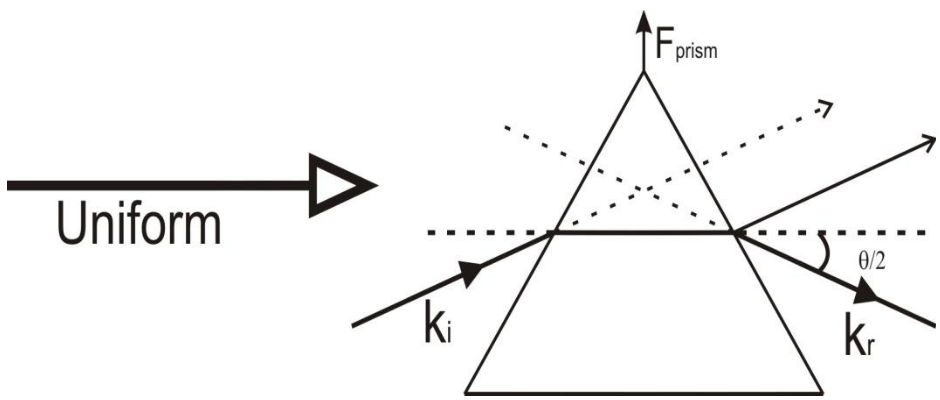
- ODF associated with in phase component of dipole moment
- refraction
- in above example, force on bead arises from intensity gradient associated with spatial profile of laser beam
- in above example ODF pulls bead toward region of high intensity



- $n_{bead} > 1$
- in above example, net force toward pushes bead toward focus
- in above example, force is associated with intensity gradient along the direction of propagation
- "axial" force is in addition to "radial" trapping force illustrated in previous example



- Note that effect of index of refraction is apparent far from ω_o whereas absorption is associated with frequencies close to the resonant frequency.



The example above further illustrates that the force on a macroscopic object (prism) is associated with the refraction of light. The index of refraction of the prism defines the refraction angle.

$$|\Delta \mathbf{p}| = \mathbf{k}_i - \mathbf{k}_r = 2k\sin(\theta/2)$$

$$\frac{F_{prism}}{A} = \left(\frac{1}{A}\right)\left(\frac{1}{\Delta t}\right)\left(\frac{energy}{c}\right)\left(2\sin\left(\frac{\theta}{2}\right)\right) = \left(\frac{I}{c}\right)\left(2\sin\left(\frac{\theta}{2}\right)\right)$$

$$\langle F_{prism} \rangle = I_{avg}\frac{A\left(2\sin\left(\frac{\theta}{2}\right)\right)}{c}$$

- angle depends on index
- F_{prism} comparable to F_{rad}

In many applications of the ODF, a relatively strong force is realized far off resonance in the near absence of photon scattering

Optical Dipole Force – Classical Model

classical model – correct frequency dependence for force
OBE model – correct intensity dependence and correct frequency dependence for force

- Dispersion/Absorption = related
- strong absorption => large $dn/d\omega$

$$qx = \varepsilon_o \chi E$$

- qx is the dipole moment
- $\varepsilon_o \chi$ is the polarizability

Interaction Energy

$$U = \tfrac{1}{2}\, \varepsilon_o \chi E^2$$

- where E is position dependent

$$F_z = -\frac{\partial U}{\partial z} = \varepsilon o \chi E \frac{dE}{dz} = qx \frac{\partial E}{\partial z} \qquad \textbf{(255)}$$

since $\mathbf{E} = E_o \cos(\omega t - kz)\hat{x}$ **(256)** (wave traveling along $\hat{z}$, polarized along $\hat{x}$)

$$F_z = -qx\left[\frac{\partial E_o(z)}{\partial z}\cos(\omega t - kz) + kE_o \sin(\omega t - kz)\right] \quad \textbf{(257)}$$

From Classical Model $x(t) = \mathcal{U}\cos(\omega t) - \mathcal{V}\sin(\omega t)$ **(234)**

- using (234) including the spatial part, in (257)
  $$F_z = q[\mathcal{U}\cos(\omega t - kz) - \mathcal{V}\sin(\omega t - kz)]\left[\frac{\partial E_o}{\partial z}\cos(\omega t - kz) + kE_o \sin(\omega t - kz)\right]$$
  (258)

$$< F_z > = \left(\frac{q2}{4m\omega}\right)\left[\frac{-\Delta E_o \frac{\partial E_o}{\partial z}}{\Delta^2 + \left(\frac{\Gamma}{2}\right)^2} + \frac{\left(\frac{\Gamma}{2}\right) k E_o{}^2}{\Delta^2 + \left(\frac{\Gamma}{2}\right)^2}\right] \quad \textbf{(260)}$$ (time averaged expression)

- use $I = \tfrac{1}{2}\, \varepsilon_o c E_o{}^2$ and generalize to 3D

$$\vec{F}_{avg} = \left(\frac{q^2}{2\varepsilon_o mc}\right)\left[\left(\frac{-\Delta}{\left[\Delta^2 + \left(\frac{\Gamma}{2}\right)^2\right]}\right)\left(\frac{\vec{\nabla} I}{\omega}\right) + \left(\frac{\frac{\Gamma}{2}}{\left(\Delta^2 + \left(\frac{\Gamma}{2}\right)^2\right]}\right)\left(\frac{I}{c}\right)\left(\frac{\vec{k}}{|k|}\right)\right] \qquad (\mathbf{261})$$

Remarks: First term in force (optical dipole force) arises from in phase component
- $1/\omega$ dependence negligible if $\Gamma << \omega_o$ (narrow resonance)
- component = 0 If $\Delta=0$

- comment on sign of force and dependence on Δ

Second term in force (radiation pressure force) arises from quadrature component

- associated with absorption → Lorenztian
- $\mathbf{F}_{rad}$ is along $\mathbf{k}$

- Classical model predicts correct frequency dependence