Physics 4062/5062 – Lecture Six – Atom Trapping Force

Outline

Physical Principle of MOT – Review

Trapping Force – Doppler Shift and Zeeman Shift

Damped Motion and Spring Constant for Restoring Force

Polarization Gradient Cooling – Physical Picture

Review Physical Principle of MOT using J = 0 → J′=1 transition

Derive Trapping Force in MOT

- Recall force due to absorption

$$F_{abs} = \left[\frac{\frac{I}{I_S}}{1+\frac{4\Delta^2}{\Gamma^2}}\right]\hbar k\Gamma \qquad \textbf{(14)}$$

$\Delta = \omega - \omega_o$

- Recall that Δ was corrected for Doppler shift
- Now, correct Δ for Doppler shift and Zeeman shift
- Recall from Lecture One:
  - Zeeman energy shift = $\mu\left(\frac{dB}{dz}\right)z = \mu_B g_F m_F\left(\frac{dB}{dz}\right)z$
  - Zeeman frequency shift = $\omega_z = \left(\frac{\mu}{\hbar}\right)\left(\frac{dB}{dz}\right) = \left(\frac{\mu_B g_F m_F}{\hbar}\right)\left(\frac{dB}{dz}\right)z = \beta z$ where $\beta = \left(\frac{\mu_B g_F m_F}{\hbar}\right)\left(\frac{dB}{dz}\right)$

$$F_{MOT} = F_+ + F_- = \left(\frac{I}{I_s}\right)\hbar k\Gamma\left[\left(\frac{1}{1+\left(\frac{4}{\Gamma^2}\right)[\Delta - kv - \beta z]^2}\right) - \left(\frac{1}{1+\left(\frac{4}{\Gamma^2}\right)[\Delta + kv + \beta z]^2}\right)\right] \qquad \textbf{(32)}$$

$$F_{MOT} = 8\left(\frac{I}{I_s}\right)\hbar k\left(\frac{\left[\frac{2\Delta}{\Gamma}\right][kv+\beta z]}{\left[1+\left(\frac{2\Delta}{\Gamma}\right)^2\right]^2}\right) \qquad \textbf{(33)}$$

- Motion is damped harmonic; equation 33 is of the form

$$F_{mot} = -\alpha v - \left(\frac{\alpha}{k}\right)\beta z = -\alpha v - k_{spring} z \qquad \textbf{(34)}$$

- The spring constant $\boxed{k_s = \frac{\alpha}{k}\beta}$ **(35)** is proportional to dB/dz

$$\omega_{trap} = \sqrt{\frac{k_{spring}}{M_{atom}}} \qquad \textbf{(36)}$$

Motion is described by equation for damped harmonic oscillator

$$\ddot{z} + \alpha\dot{z} + \omega_{trap}{}^2 z = 0 \qquad \textbf{(37)}$$

$$\alpha = \frac{8\hbar k^2\left(\frac{I}{I_S}\right)\left(\frac{2\Delta}{\Gamma}\right)}{M_{atom}\left[1+\left(\frac{2\Delta}{\Gamma}\right)^2\right]^2}$$

$$\omega_{trap}^2 = \frac{8\hbar k\left(\frac{I}{I_s}\right)\left(\frac{2\Delta}{\Gamma}\right)\beta}{M_{atom}\left[1+\left(\frac{2\Delta}{\Gamma}\right)^2\right]^2} \quad \textbf{(38)}$$

- if $\alpha^2/4\omega_{trap}^2 > 1$ => overdamped motion

Motion is described by

$$z = A \exp[-k_s t/\alpha]$$

- α/k_{spring} ~ 1ms for dB/dz ~ 10 G/cm (typical gradient)
- estimate ω_{trap}
- estimate well depth