Phys 4062/5062 – Lecture Seven – Atom Field Interactions

Outline

Atom Field Interaction – Dipole Approximation

Time Dependent Evolution of Probability Amplitudes for Weak Fields

Atomic Response for Pulsed Excitation

Atom Field Interaction

Semi-Classical Treatment

- Electric field is classical
- Atom is a quantum mechanical entity

Solve: $i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$ **(201)**

$$\hat{H} = \widehat{H_o} + \widehat{H_I}(t)$$

- H_o is the unperturbed Hamiltonian
  - eigenvalues – energy levels
  - eigenfunctions – wavefunctions
    $\psi_n(r,t) = \psi_n(r)\exp[-iE_n t/\hbar]$ **(202)**
  - ψ_n retains character of wavefunctions of H_o such as parity and satisfies
    $\boxed{H_o\psi_n(r) = E_n\psi_n(r)}$
- H_I is the interaction of Electric Field that perturbs eigenfunctions of H_o

External Perturbation

$\mathbf{E} = \mathbf{E_o}\cos(\omega t) = |\mathbf{E_o}|\cos(\omega t)\hat{x}$ (Linearly polarized field)

$H_I = q\mathbf{r}\cdot\mathbf{E_o}\cos(\omega t) = \mathbf{p}\cdot\mathbf{E}$

$\omega_o = (E_2 - E_1)/\hbar$ (Resonant Frequency)

Introduce Rabi Frequency

$\Omega = \frac{qX_{12}|E_o|}{\hbar}$ where

$<X_{12}> = <1 \,|\, \hat{x} \,|\, 2>$

$\psi(r,t) = c_1\psi_1(r)e^{-\frac{iE_1 t}{\hbar}} + c_2\psi_2(r)e^{-\frac{iE_2 t}{\hbar}}$ (Linear Superposition of Unperturbed States)

$\psi(r,t) = c_1|1\rangle e^{-\frac{i\omega_1 t}{\hbar}} + c_2|2\rangle e^{-\frac{i\omega_2 t}{\hbar}}$ **(204)**

Here the probability amplitudes satifies the normalization condition

$|c_1(t)|^2 + |c_2(t)|^2 = 1$ **(205)**

Using (204)and (201)

$i\dot{c}_1 = \Omega\cos(\omega t)\, e^{-i\omega_o t}c_2$ **(207)**

$i\dot{c}_2 = \Omega^*\cos(\omega t)\, e^{i\omega_o t}c_1$

Assume Initial Conditions

$c_1(0) = 1$
$c_2(0) = 0$
$t << T \Rightarrow |c_2(t)^2| << 1$

- there are weak field conditions

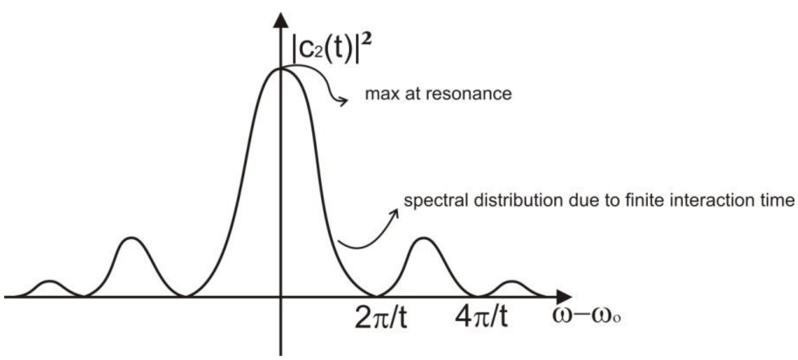
- T is the interaction time
- Integrate (207) to get

$$c_2(t) = \left(\frac{\Omega}{2}\right)\left[\frac{1-\exp[i(\omega_o+\omega)t]}{\omega_o+\omega} + \frac{1-\exp[i(\omega_o-\omega)t]}{\omega_o-\omega}\right] \qquad \textbf{(210)}$$

- for optical frequencies $\omega \sim \omega_o$
- $|\Delta| = |\omega_o - \omega| << \omega_o$
- $\omega_o + \omega \sim 2\omega_o$
- this is called the rotating wave approximation which results in elimination of 1st term in (210)

Probability of Atom in |2> at time t is given by

$$|c_2(t)|^2 = |\Omega|^2\left[\frac{\sin\left(\frac{[\omega_o-\omega]t}{2}\right)}{\omega_o-\omega}\right]^2 \qquad \textbf{(211)}$$



- analogous to single diffraction pattern
- spread decreases when interaction time t increases just like diffraction angle decreases when slit width increases
- contrast this behaviour to steady state response
- relate atomic response to spectral components (Fourier components) associated with a square pulse